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Massive String Cloud Cosmologies in Saez-Ballester Theory of Gravitation

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Abstract String cloud cosmological models are studied using spatially homogeneous and anisotropic Bianchi type VI_0 metric in Saez-Ballester Scalar-Tensor theory of gravitation. The field equations are solved for massive string cloud with particles attached to them. A more general linear equation of state of the cosmic string tension density with the proper energy density of the universe is considered instead of taking any particular relationships like pure geometric string or the case of the p-string. The pure geometric string and p-string solutions can be easily inferred from the models. For all viable models the possible limiting values of the linear connection between the proper energy density and string tension density have been calculated. The physical and kinematical properties of the models have been discussed in detail.

Keywords String cosmology · Saez-Ballester theory

1 Introduction

In recent years there has been a lot of academic and research interest on modified theories of gravity because of their ability to predict the cosmic acceleration [29] without the need for sources of dark energy. In general relativity the observed accelerated expansion of the

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late time universe predicted from a lot of data like type Ia supernova (SN-Ia), the cosmic microwave background radiation and the large scale structure of the universe is usually explained by Friedman-Robertson-Walker (FRW) models with a cosmological constant Λ. In these sense the Scalar-Tensor theories developed with a motivation to incorporate a time varying Newtonian gravitational constant 'G' have generated much research interest. Brans and Dicke's theory of gravitation [8] with the gravity being mediated by a long range scalar field and Saez-Ballester's Scalar-Tensor theory [22] have been widely used to describe the evolution of the early universe. In Saez-Ballester's theory of gravitation, the metric is coupled to a scalar field that satisfactorily describes the weak fields and removes the missing matter problems in the non-flat FRW cosmologies. The Scalar-Tensor theories of gravitation play an important role to remove the graceful exit problem in the inflationary era [14]. The high degree of isotropy of the 2.7 K microwave background radiation supports the FRW models. However certain measurements have indicated dipole anisotropy of the microwave back ground radiation [7]. While the dipole anisotropy can readily be explained by the Sun's motion relative to the background radiation, the quadurpole anisotropy is an intrinsic property of the background radiation. Since the experimental data favours an anisotropic universe, it is wise to consider spatially homogeneous and anisotropic Bianchi type cosmological models. These models have a significant role in the description of the universe at the early stages of evolution. Of late, there has been a lot of interest in Bianchi type models. Mohanty and Sahu [12, 13], Reddy and Rao [19], Singh and Chaubey [26], Shanti and Rao [24], Rao and Sanyasiraju [18] have studied the Bianchi type universes in Scalar-Tensor theories. Different Bianchi type universes have also been studied in the framework of Einstein Relativity with a cosmological constant [25].

Cosmic strings are believed to be some one dimensional topological stable defects during the phase transition of the early universe among others like monopole and domain walls. Out of these topological defects, strings can lead to many interesting effects. The density perturbation arising out of them leads to the formation of galaxies [31]. The description of the evolution of the universe in the presence of cloud of cosmic string dust has generated a lot of interest in recent times. Letelier [11], Stachel [28], Bali et al. [2–4], Yadav et al. [35, 36], Pradhan et al. [15, 16], Wang [32–34], Baysal et al. [6] have investigated different aspects of cosmic string in general relativity. Carminati et al. [9], Banerjee et al. [5], Soares et al. [27] have studied the effect of magnetic field on string cosmological models. Katore and Rane [10] have discussed Bianchi-III cosmologies in presence of cosmic string dust and electromagnetic field in Rosen's Bimetric theory. In Rosen's Bimetric theory, Sahoo [23] has studied string cosmology in spherically symmetric space-time. Reddy et al. [20, 21], Rahaman et al. [17] have studied string cosmological models in modified theories of gravitation in higher dimensions. In a recent paper [30], we have studied the effect of magnetic field on Bianchi type-III geometric string cloud cosmological models in the frame work of Saez-Ballester's theory of gravitation. In the frame work of general relativity, Bali et al. [4] have studied the magnetized barotropic massive cosmic string fluid models for Bianchi type VI_0 universe. Since the universe can be represented by a collection of extended objects like galaxies, string cloud cosmological models can help understand the evolution of the early universe.

The purpose of this paper is to study spatially homogeneous and anisotropic Bianchi type VI_0 universe in the scalar-tensor theory as proposed by Saez and Ballester [22] in the presence of cloud of cosmic string dust satisfying a more general linear relationship with the gravitational field. The organization of the paper is as follows. In Sect. 2, starting from a Bianchi VI_0 model, the field equations in presence of cosmic string cloud are derived. In Sect. 3, the consequences of the field equations are discussed for the energy density satisfying a general linear equation of state. In Sect. 4, different possible cosmological models for massive string cloud in the Scalar Tensor theory are presented and the physical and kinematical properties of the models are discussed. At the end the conclusion of the discussed models are presented in the Sect. 5.

2 Field Equations for Massive Cosmic String Cloud Model

The Bianchi type VI_h line element is considered in the form [13]

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2hx} dy^2 + C^2 e^{2hx} dz^2 \tag{1}$$

where A, B, C are the metric potentials considered as functions of cosmic time only. 'h' is a chosen exponent to describe the model.

In a combined Scalar-Tensor field, the Saez-Ballester field equations are given by

$$R_{ij} - \frac{1}{2} g_{ij} R - w\phi^n \left(\phi_{,i}\phi_{,j} - \frac{1}{2} g_{ij} \phi_{,a}\phi^{,a} \right) = -T_{ij} \tag{2}$$

and the scalar field ϕ satisfies the equation

$$2\phi^n \phi^{;i}_{,i} + n\phi^{n-1}\phi_{,a}\phi^{,a} = 0 \tag{3}$$

where R_{ij} is the Ricci tensor and R is Ricci Scalar, n is an arbitrary exponent and w is a dimensionless coupling constant. Here the comma (,) after the scalar field denotes ordinary differentiation. The energy-momentum tensor for a cloud of cosmic string is given by

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \tag{4}$$

where ρ is the rest energy density of the system, λ is the string tension density. λ can be positive or negative [11]. The strings are assumed to be massive strings and particles are attached to them. The energy density of the particles is taken to be $\rho_p = \rho - \lambda$. u^i is the four-velocity vector and x^i, the space-like vector represents the anisotropic direction of the string. u^i and x^i satisfy the equations,

$$g_{ij}u^i u^j = -1 \tag{5a}$$

$$g_{ij}x^i x^j = 1 \quad \text{and} \tag{5b}$$

$$u^i x_i = 0 \tag{5c}$$

Choosing x^i parallel to $\partial/\partial x$, we have

$$x^i = (A^{-1}, 0, 0, 0) \tag{6}$$

From the conservation law,

$$T^{ij}_{;j} = 0 \tag{7}$$

where the semicolon denotes covariant derivative of the energy-momentum tensor. With the help of (4)–(7), the field equations (2) and (3), for the Bianchi type VI_0 metric can take the explicit forms:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{2}w\phi^n\dot{\phi}^2 = \lambda \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{2}w\phi^n\dot{\phi}^2 = 0 \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{2}w\phi^n\dot{\phi}^2 = 0 \tag{10}$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{11}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} + \frac{1}{2}w\phi^n\dot{\phi}^2 = \rho \tag{12}$$

$$(\rho - \lambda)\frac{\dot{A}}{A} + \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\rho = -\dot{\rho} \tag{13}$$

$$\ddot{\phi} + \frac{n}{2}\frac{\dot{\phi}^2}{\phi} = 0 \tag{14}$$

The overhead dots represent time derivatives and double dots represent double differentiation with respect to time. Here we have chosen the units $8\pi G = c = 1$, where c is the speed of light in free space. The scalar field ϕ incorporates the time varying nature of Newtonian Gravitational constant.

3 Consequences of the Field Equations

Equation (11) immediately suggests

$$B = kC \tag{15}$$

with $k > 0$ an integration constant.

With equation (15), the field equations (8)–(13) reduce to

$$2\frac{\ddot{C}}{C} + \left(\frac{\dot{C}}{C}\right)^2 - \frac{1}{2}w\phi^n\dot{\phi}^2 = \lambda \tag{16}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{2}w\phi^n\dot{\phi}^2 = 0 \tag{17}$$

$$2\frac{\dot{C}\dot{A}}{CA} + \left(\frac{\dot{C}}{C}\right)^2 + \frac{1}{2}w\phi^n\dot{\phi}^2 = \rho \tag{18}$$

$$(\rho - \lambda)\frac{\dot{A}}{A} + 2\rho\frac{\dot{C}}{C} = -\dot{\rho} \tag{19}$$

Since the field equations are highly nonlinear in nature the solutions are tried for a more general linear equation of state

$$\rho = \gamma\lambda \tag{20}$$

where γ describes the relationship between ρ and λ. The prevailing string equations of state (EOS) like the geometric string EOS or $\rho = -\lambda$ and the p-string or Takabayasi string EOS $\rho = (1+W)\lambda$ with $W > 0$ can be easily inferred from this generalized EOS.

With this EOS the energy density arising from the particles assumes the form

$$\rho_p = (\gamma - 1)\lambda \tag{21}$$

Using (20) in (19) we get

$$\lambda = \frac{\xi_0}{A^{(\frac{\gamma-1}{\gamma})} C^2} \tag{22}$$

where $\xi_0 \neq 0$ is a constant of integration.

Equation (14) on integration yields

$$\phi^{\frac{n}{2}+1} = \left(\frac{n}{2} + 1\right)(m_1 t + m_2) \tag{23}$$

which in turn leads to

$$\frac{1}{2} w \phi^n \dot{\phi}^2 = \frac{1}{2} w m_1^2 = \mu = \text{constant} \tag{24}$$

where $m_1 \neq 0$, m_2 being the integration constants.

Defining $\alpha = \frac{\dot{A}}{A}$ and $\beta = \frac{\dot{C}}{C}$, from (16)–(18), (20) and (24) we get for $\gamma \neq 0$ ($\gamma = 0$ represents a vacuum universe with $\rho = 0$)

$$(\dot{\beta} - s) + \beta(p\beta - q\alpha) = 0 \tag{25}$$

where

$$p = \frac{3\gamma - 1}{2\gamma} \tag{26a}$$

$$q = \frac{1}{\gamma} \tag{26b}$$

and

$$s = \left(\frac{\gamma + 1}{2\gamma}\right)\mu \tag{26c}$$

Equation (25) can be satisfied for the following three different plausible considerations:

$$\text{Case-I:} \quad \dot{\beta} = s, \qquad \beta = 0 \tag{27}$$

$$\text{Case-II:} \quad \dot{\beta} = s, \qquad \beta = \frac{q}{p}\alpha \tag{28}$$

$$\text{Case-III:} \quad \dot{\beta} = s - p\beta^2 + q\alpha\beta \tag{29}$$

Consequently, we can have three different cases of solution to the Saez-Ballester field equations.

4 Cosmological Models for Massive Cosmic String Cloud

4.1 Case-I

Equation (27) suggests that

$$s = \left(\frac{\gamma + 1}{2\gamma}\right)\mu = 0 \tag{30}$$

For $\mu = 0$, the Saez-Ballester theory is either stripped of its Scalar field coupling to general relativity or the scalar field be a constant of time, both of which are non interesting for a Scalar-Tensor theory. Therefore for Saez-Ballester theory $\mu \neq 0$. Thus (30) leads to $\gamma = -1$ which implies $\rho = -\lambda$. Since for viable cosmological models $\rho > 0$, this result implies $\lambda < 0$. A negative string tension density λ indicates that string phase of the universe should disappear and the universe be filled with an anisotropic fluid of particles.

For this case $\rho + \lambda = 0$ and we have

$$B = m_3 = \text{constant} \tag{31}$$

$$C = m_4 = \text{constant} \tag{32}$$

With $\beta = 0$ and $\alpha = \frac{\dot{A}}{A}$ we get from (17)

$$\dot{\alpha} + \alpha^2 - \mu = 0 \tag{33}$$

which can be solved for two different nature of μ i.e. for $\mu > 0$ and $\mu < 0$.

Case-(a): For $\mu > 0$: With μ having only non-zero positive values, integration of (33) yields

$$\alpha = \sqrt{\mu} + \frac{2\xi_1\sqrt{\mu}\exp(-2\sqrt{\mu}t)}{1 - \xi_1\exp(-2\sqrt{\mu}t)} \tag{34}$$

and successive integration of (34) gives

$$A = \xi_2(1 - \xi_1\exp(\sqrt{\mu}t))e^{\sqrt{\mu}t} \tag{35}$$

where $\xi_1 > 0$, $\xi_2 > 0$ are integration constants. With suitably chosen values for the integration constants we can have

$$A = 2\sinh(\sqrt{\mu}t) \tag{36}$$

The metric for the model can be written as

$$ds^2 = -dt^2 + \xi_2^2(1 - \xi_1\exp(-2\sqrt{\mu}t))^2 e^{2\sqrt{\mu}t}dx^2 + m_3^2dy^2 + m_4^2dz^2 \tag{37}$$

and with proper choice of constants and coordinates

$$ds^2 = -dT^2 + (\sinh^2 T)dX^2 + dY^2 + dZ^2 \tag{38}$$

The physical and kinematical parameters of the model are

$$\rho = -\lambda = \frac{-\lambda_0}{\sinh^2 T} \tag{39}$$

$$\text{Particle density:} \quad \rho_p = -2\lambda = \frac{-2\lambda_0}{\sinh^2 T} \tag{40}$$

$$\text{The scalar expansion:} \quad \theta = \coth T \tag{41}$$

$$\text{The shear scalar:} \quad \sigma^2 = 4\theta^2 = 4\coth^2 T \quad \text{where } \lambda_0 \text{ is a constant} \tag{42}$$

In the beginning, when $T \to 0$, the proper energy density, the string tension density and the particle density approach large values. The singularity also occurs for the scalar expansion and shear scalar. The physical and kinematical parameters of the model vary harmonically with the growth of cosmic time. But the model is in general not isotropic as $\frac{\sigma}{\theta} \neq 0$. In the model, the realistic energy condition $\rho \geq 0$ can be satisfied only if $\lambda_0 < 0$.

Case-(b): *For* $\mu < 0$: When μ assumes negative values, with a newly defined constant $\nu = -\mu$, the integration of (33) yields

$$\alpha = \sqrt{\nu}\tan(-\sqrt{\nu}t + \xi_3\sqrt{\nu}) \tag{43}$$

which in turn gives

$$A = \left[\xi_4 \sec(-\sqrt{\nu}t + \xi_3\sqrt{\nu}\,\right]^{\sqrt{\nu}} \tag{44}$$

where ξ_3 and $\xi_4 \neq 0$ are constants of integration.

The metric for this model can be written as

$$ds^2 = -dt^2 + \left[\xi_4 \sec(-\sqrt{\nu}t + \xi_3\sqrt{\nu})\right]^{2\sqrt{\nu}} dx^2 + m_3^2 dy^2 + m_4^2 dz^2 \tag{45}$$

and with proper choice of coordinates and constants, the metric reduces to

$$ds^2 = -dT^2 + \sec^2(1-T)dX^2 + dY^2 + dZ^2 \tag{46}$$

The physical and kinematical parameters of the model are

$$\rho = -\lambda = \frac{-\lambda_0}{\sec^2(1-T)} = -\lambda_0\cos^2(1-T) \tag{47}$$

$$\rho_p = -2\lambda = -2\lambda_0\cos^2(1-T) \tag{48}$$

$$\theta = \tan(1-T) \tag{49}$$

$$\sigma^2 = 4\tan^2(1-T) \tag{50}$$

When $T \to 0$, ρ and λ assume some constant values and with the advancement of the arrow of cosmic time they vary in accordance with harmonic functions. Also in this model, the realistic energy condition $\rho \geq 0$ can be satisfied only if $\lambda_0 < 0$. As we have discussed earlier, the model (38) and (46) are not viable string cosmological models since with the choice of $\gamma = -1$, the string phase of the universe disappears. The discussions here are made only out of an academic interest to show that with $\rho + \lambda = 0$, the Saez-Ballester theory for Bianchi type-VI$_0$ metric provides unphysical results.

4.2 Case-II

On integration, the first part of the (28) i.e. $\dot{\beta} = s$, yields

$$C = \xi_5 \exp\left(\frac{st^2}{2} + \xi_6 t\right) = \xi_5 \exp\left\{\left(\frac{\gamma+1}{4\gamma}\right)\mu t^2 + \xi_6 t\right\} \quad \text{and} \tag{51}$$

$$B = k\xi_5 \exp\left(\frac{st^2}{2} + \xi_6 t\right) = k\xi_5 \exp\left\{\left(\frac{\gamma+1}{4\gamma}\right)\mu t^2 + \xi_6 t\right\} \tag{52}$$

where $\xi_5 \neq 0$ and ξ_6 are constants.

Since $\beta = \frac{q}{p}\alpha$ or $\alpha = (\frac{3\gamma-1}{2})\beta$, we can have

$$\begin{aligned} &A = A_0 C^{(\frac{3\gamma-1}{2})} \quad \text{or} \\ &A = A_1 \exp\left[\left\{\left(\frac{\gamma+1}{4\gamma}\right)\mu t^2 + \xi_6 t\right\}\left(\frac{3\gamma-1}{2}\right)\right] \end{aligned} \tag{53}$$

where A_0, A_1 are finite non-zero constants.

The metric for the model assumes the form

$$\begin{aligned} ds^2 = {} & -dt^2 + A_1^2 \exp\left[\left\{\left(\frac{\gamma+1}{4\gamma}\right)\mu t^2 + \xi_6 t\right\}(3\gamma-1)\right] dx^2 \\ & + \xi_5^2 \exp\left[2\left(\left(\frac{\gamma+1}{4\gamma}\right)\mu t^2 + \xi_6 t\right)\right]\left[k^2 dy^2 + dz^2\right] \end{aligned} \tag{54}$$

which can be reduced to

$$\begin{aligned} ds^2 = {} & -dT^2 + \exp\left[\left\{\left(\frac{\gamma+1}{4\gamma}\right)T^2 + T\right\}(3\gamma-1)\right] dX^2 \\ & + \exp\left[\left(\frac{\gamma+1}{2\gamma}\right)T^2 + 2T\right]\left[dY^2 + dZ^2\right] \end{aligned} \tag{55}$$

The physical and geometrical properties for the model are

$$\lambda = \frac{\rho}{\gamma} = \frac{\lambda_1}{\exp[[(\frac{\gamma+1}{2\gamma})T^2 + 2T](\frac{3\gamma^2+1}{4\gamma})]} \tag{56}$$

$$\rho_p = \frac{(\gamma-1)\lambda_1}{\exp[[(\frac{\gamma+1}{2\gamma})T^2 + 2T](\frac{3\gamma^2+1}{4\gamma})]} \tag{57}$$

where λ_1 is a constant

$$\theta = \frac{3}{4\gamma}(\gamma+1)^2 T + \frac{3}{2}(\gamma+1) \tag{58}$$

$$\sigma^2 = \frac{1}{4}(27\gamma^2 + 42\gamma + 26)\left[\left(\frac{\gamma+1}{2\gamma}\right)^2 T^2 + \left(\frac{\gamma+1}{\gamma}\right)T + 1\right] \tag{59}$$

For viable string cosmological models $\rho \geq 0$, $\rho_p \geq 0$ which suggest that $\gamma > 0$ and in fact for non-geometric cosmic string γ should be greater than one as in p-string or Takabayasi string. For $\gamma > 0$, the universe starts with (for $T \to 0$)

$$\lambda = \frac{\rho}{\gamma} = \frac{\rho_p}{\gamma - 1} = \lambda_1$$

But with the growth of time, the string tension density, the proper energy density and the particle density asymptotically reach to null values as $T \to \infty$.

For pure geometric string, the physical and kinematical parameters become

$$\rho = \lambda = \frac{\lambda_1}{\exp(T^2 + 2T)} \tag{60a}$$

$$\rho_p = 0 \tag{60b}$$

$$\theta = 3(T + 1) \tag{60c}$$

For p-string with $\rho = (1 + W)\lambda$ and $W > 0$,

$$\lambda = \frac{\lambda_1}{\exp[[(\frac{2+W}{2(1+W)})T^2 + 2T](\frac{3W^2+6W+4}{4(1+W)})]} \tag{61a}$$

$$\rho = \frac{(1 + W)\lambda_1}{\exp[[(\frac{2+W}{2(1+W)})T^2 + 2T](\frac{3W^2+6W+4}{4(1+W)})]} \tag{61b}$$

$$\rho_p = \frac{W\lambda_1}{\exp[[(\frac{2+W}{2(1+W)})T^2 + 2T](\frac{3W^2+6W+4}{4(1+W)})]} \tag{61c}$$

Small value of W represents a string dominated universe where as it requires a large value to represent a particle dominated universe at large cosmic time. For viable cosmological models in this category, $\gamma > 0$ implies $\theta > 0$ which indicates that the model is expanding in nature. For all real values of γ, $\frac{\sigma}{\theta} \neq 0$ implying that the model is not isotropic for whole range of cosmic time.

4.3 Case-III

In order to get some plausible solutions for (29) we assume $\alpha = \eta\beta$ as has been taken earlier by Bali [1] and Katore et al. [10] so that $A = (BC)^{\eta/2}$, where η is a chosen exponent. This is in accordance with the fact that the expansion scalar θ be proportional to the shear scalar σ. With this assumption (29) reduces to

$$\frac{\dot{\beta}}{\beta^2 - \frac{s}{p-\eta q}} = -(p - \eta q) \tag{62}$$

which can have two different solutions for the two possibilities $\frac{s}{p-\eta q} > 0$ and $\frac{s}{p-\eta q} < 0$, provided $p \neq \eta q$.

Case (a): $\frac{s}{p-\eta q} > 0$ *For this case, integration of (62) yields*

$$\beta = \sqrt{\frac{s}{p - \eta q}} + \frac{2k_1\sqrt{\frac{s}{p-\eta q}}\exp[-2t\sqrt{s(p - \eta q})]}{1 - k_1\exp[-2t\sqrt{s(p - \eta q)}]} \tag{63}$$

which on further integration leads to

$$C = k_2[1 - k_1\exp[-2t\sqrt{s(p - \eta q)}]]^{\frac{1}{p-\eta q}}\exp\left(t\sqrt{\frac{s}{p - \eta q}}\right) \tag{64}$$

Since $\frac{s}{p-\eta q} = \frac{(\gamma+1)\mu}{3\gamma-1-2\eta}$

$$C = k_2 \left(1 - k_1 \exp\left[-\frac{t}{\gamma}\sqrt{\mu\left(3\gamma^2 + 2\gamma(1-\eta) - (1+2\eta)\right)}\right]\right)^{\frac{2\gamma}{3\gamma-1-2\eta}} \times \exp\left[t\sqrt{\frac{(\gamma+1)\mu}{3\gamma-1-2\eta}}\right] \tag{65}$$

and

$$B = kC = kk_2\left(1 - k_1 \exp\left[-\frac{t}{\gamma}\sqrt{\mu(3\gamma^2 + 2\gamma(1-\eta) - (1+2\eta))}\right]\right)^{\frac{2\gamma}{3\gamma-1-2\eta}} \times \exp\left[t\sqrt{\frac{(\gamma+1)\mu}{3\gamma-1-2\eta}}\right] \tag{66}$$

$$A = k_3\left(1 - k_1 \exp\left[-\frac{t}{\gamma}\sqrt{\mu(3\gamma^2 + 2\gamma(1-\eta) - (1+2\eta))}\right]\right)^{\frac{2\eta\gamma}{3\gamma-1-2\eta}} \times \exp\left[\eta t\sqrt{\frac{(\gamma+1)\mu}{3\gamma-1-2\eta}}\right] \tag{67}$$

where k_1, k_2, k_3 are constants.

The physical and kinematical aspects of the model are

$$\lambda = \frac{\rho}{\gamma} = \frac{\rho_p}{\gamma - 1} = \frac{\lambda_3}{[(1 - k_1 \exp\{-\frac{t}{\gamma}\sqrt{\mu(3\gamma^2 + 2\gamma(1-\eta) - (1+2\eta))}\})^{\frac{2\gamma}{3\gamma-1-2\eta}} \exp(t\sqrt{\frac{(\gamma+1)\mu}{3\gamma-1-2\eta}})]^{2+\frac{\eta}{\gamma}(\gamma-1)}} \tag{68}$$

$$\theta = (\eta + 2)\beta \tag{69}$$

$$\sigma^2 = 4\theta^2 \tag{70}$$

where is λ_3 is a constant and β is given by (63).

It is amply clear from the physical parameters of the model that for a sustainable model $\gamma > (1+2\eta)/3$ for all positive values of μ and γ. We can not have a non-singular model with $\gamma = 1$ and $\eta = 1$. For physical p-string solutions $\eta < (1 + \frac{3W}{2})$ and for pure geometric string case, η should be less than 1. Besides these restrictions, when $T \to 0, \lambda, \rho$ and ρ_p have some finite non-zero values such as $\lambda_3, \gamma\lambda_3$ and $(\gamma - 1)\lambda_3$ respectively. For $k_1 = 1$, there occurs a point type singularity at $T \to 0$ for the physical properties λ, ρ, ρ_p. When $T \to \infty, \lambda, \rho$ and ρ_p identically vanish. The model is in general anisotropic for the whole range of cosmic time.

The realistic energy condition $\rho \geq 0$ and $\rho_p \geq 0$ can be satisfied if $\lambda_3 > 0$, $\gamma > 0$ and $\frac{\eta}{\gamma}(\gamma - 1) = 2\chi$ where χ is an integer. In the other words, the realistic energy condition is satisfied if $\gamma = \frac{1}{\eta - 2\chi}$.

Case (b): $\frac{s}{p-\eta q} < 0$ *For this case (62) can be written as*

$$\frac{\dot{\beta}}{\beta^2 + \frac{s}{\eta q - p}} = (\eta q - p) \tag{71}$$

which on integration gives

$$\beta = \sqrt{\frac{s}{\eta q - p}} \tan\left(t\sqrt{s(\eta q - p)} + k_4\sqrt{\frac{s}{\eta q - p}}\right) \tag{72}$$

and with a successive integration yields

$$C = \left[k_5 \sec\left(t\sqrt{s\,(\eta q - p)} + k_4\sqrt{\frac{s}{\eta q - p}}\right)\right]^{\sqrt{\frac{s}{\eta q - p}}} \tag{73}$$

substitution of the values of s, q, p results in

$$C = \left[k_5 \sec\left(\frac{t}{2\gamma}\sqrt{\mu((2\eta+1) + 2\gamma(\eta-1) - 3\gamma^2)} + k_4\sqrt{\frac{(\gamma+1)\mu}{2\eta+1-3\gamma}}\right)\right]^{\sqrt{\frac{(\gamma+1)\mu}{2\eta+1-3\gamma}}} \tag{74}$$

$$B = kC = k\left[k_5 \sec\left(\frac{t}{2\gamma}\sqrt{\mu((2\eta+1) + 2\gamma(\eta-1) - 3\gamma^2)} + k_4\sqrt{\frac{(\gamma+1)\mu}{2\eta+1-3\gamma}}\right)\right]^{\sqrt{\frac{(\gamma+1)\mu}{2\eta+1-3\gamma}}} \tag{75}$$

$$A = k_6\left[\sec\left(\frac{t}{2\gamma}\sqrt{\mu((2\eta+1) + 2\gamma(\eta-1) - 3\gamma^2)} + k_4\sqrt{\frac{(\gamma+1)\mu}{2\eta+1-3\gamma}}\right)\right]^{\eta\sqrt{\frac{(\gamma+1)\mu}{2\eta+1-3\gamma}}} \tag{76}$$

where k_4, k_5, k_6 are constants.

The physical and kinematical aspects of the model are

$$\begin{aligned}\lambda &= \frac{\rho}{\gamma} = \frac{\rho_p}{\gamma - 1}\\ &= \frac{\lambda_4}{[\sec(\frac{t}{2\gamma}\sqrt{\mu((2\eta+1) + 2\gamma(\eta-1) - 3\gamma^2)} + k_4\sqrt{\frac{(\gamma+1)\mu}{2\eta+1-3\gamma}})]^{\{2+\frac{\eta}{\gamma}(\gamma-1)\}\sqrt{\frac{(\gamma+1)\mu}{2\eta+1-3\gamma}}}}\end{aligned} \tag{77}$$

where λ_4 is a constant

$$\begin{aligned}\theta &= (\eta+2)\\ &\quad\times\left[\sqrt{\frac{(\gamma+1)\mu}{2\eta+1-3\gamma}} \tan\left(\frac{t}{2\gamma}\sqrt{\mu((2\eta+1) + 2\gamma(\eta-1) - 3\gamma^2)} + k_4\sqrt{\frac{(\gamma+1)\mu}{2\eta+1-3\gamma}}\right)\right]\end{aligned} \tag{78}$$

and

$$\sigma^2 = 4\theta^2 \tag{79}$$

The model can have physical implications for $\gamma < (1+2\eta)/3$ for positive values of μ and γ. Non-singular model can not be achieved with $\gamma = 1$, $\eta = 1$. For p-string solutions, $\eta > (1 + \frac{3W}{2})$. Also for pure geometric string models ηshould be greater than 1. η is usually taken to be 1 or 2 as has been considered earlier by some authors [1, 10]. But it is amply clear from the discussion of the model that values of η less than 1 cannot lead to plausible solutions for p-string model. In this model the realistic energy condition $\rho \geq 0$, $\rho_p \geq 0$ can be satisfied if $\lambda_4 > 0$, $\gamma > 0$ and $\{2 + \frac{\eta}{\gamma}(\gamma - 1)\}\sqrt{\frac{(\gamma+1)\mu}{2\eta+1-3\gamma}} = 2\chi$ where χ is an integer.

5 Conclusion

The string cloud cosmological solutions for spatially homogeneous and anisotropic Bianchi type VI_0 model are derived from Saez-Ballester field equations. Particles are assumed to be attached with the strings forming the world sheet. The string is coupled to the usual gravitational field. In order to solve the Saez-Ballester field equations we have used a more general equation of state for the proper energy density and string tension density from which the prevailing pure geometric cosmic string and the p-string or Takabayasi string solutions can be easily inferred. In the discussion, we have avoided the possible vacuum solutions. Among the three different cases of cosmic string cloud models discussed, a physical model cannot be achieved when the sum total of the string tension density and the energy density vanishes. All other possible cases have been studied in detail along with their physical and kinematical aspects. As a general linear EOS is considered here to study the cosmological string cloud model, the possible limiting values for the ratio of the proper energy density and string tension density have been calculated. In the Case-III(b) considered, it is shown that for plausible Takabayasi string solution in Saez-Ballester theory η should be greater than one instead of 2 or 1 as usually considered. In both the model of Case-III, physical solutions can not be obtained for pure geometric string with $\rho = \lambda$ when the chosen exponent η is equal to 1. In all possible models the physical parameters evolve with time and they identically vanish for an infinitely large cosmic time.

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